Importance Sampling of Glittering BSDFs based on Finite Mixture Distributions (Supplemental Material 1/2) Convergence Comparisons

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1 Protocol

1.1 Convergence test

We investigate the convergence of the importance sampling procedure and use a Monte Carlo (MC) estimator with importance sampling to solve the equation

$$\int_{\Omega} f(\omega_o, \omega_i)|\omega_i \cdot \omega_g| \mathrm{d}\omega_i, \tag{1}$$

which corresponds to a white environment, i.e., $L(\omega_i) = 1$, as in the white furnace test. The exact value of this integral is the average value of the shadowing term G_1 ($0 \leq G_1 \leq 1$).

1.2 Raw data

We realise many estimations of Equation 1, i.e. many realisations r of the MC estimator. We collect each estimation as the number of samples N increases. Our raw data is thus a set of curves

$$F_r(N; \theta_o, \alpha, K) = \frac{1}{N} \sum_{j=1}^{N} \frac{f(\omega_o, \omega_{i_j})|\omega_{i_j} \cdot \omega_g|}{\mathrm{PDF}(\omega_{i_j})}, \tag{2}$$

where

- N is the number of samples,
- r is the index of a realisation, i.e. one estimation / one random seed,
- θ_o is the incidence angle corresponding to ω_o,
- K is the number of microfacets in the footprint,
- $\mathrm{PDF}(\omega_i)$ is the distribution used for sampling the incident direction ω_i. In the graphs below, green curves are obtained by sampling the multi-lobe component of the BSDF (our method), while red curves are obtained by sampling the mono-lobe approximation of the BSDF (previous method), namely the limit of $f(\omega_o, \omega_i)$ as $K \to \infty$.

1.3 Parameters

- $1 \leq r \leq 1,000$ realisations.
- $1 \leq N \leq 10,000$ samples.
- $\theta_o \in \{0\ ,\ 1.\ ,\ 1.5\}$
- $\alpha \in \{0.1\ ,\ 0.25\ ,\ 0.6\}$
- $K \in \{15\ ,\ 148\ ,\ 2,379\ ,\ 41,624\ ,\ 166,496\}$

1.4 Pointwise boxplot

Each graph plots the estimator against N, for a fixed set of parameters θ_o, α, K. We use semi-log graphs because of the wide range for N. Plotting F_r for all realisations r would be illegible. Aiming at a statistically more representative plot, we use pointwise boxplots, i.e., we draw curves corresponding to pointwise quartiles:

- $F_{0\%}$ and $F_{100\%}$ are the minimum and maximum (dotted lines in our graphs),
- $F_{50\%}$ is the median (solid lines in our graphs),
- $F_{25\%}$ and $F_{75\%}$ are the first and third quartile (dashed lines in our graphs).

This means that, for any fixed N, 50% of the curves F_r are such that $F_{25\%}(N) \leq F_r(N) \leq F_{75\%}(N)$.

2 Results

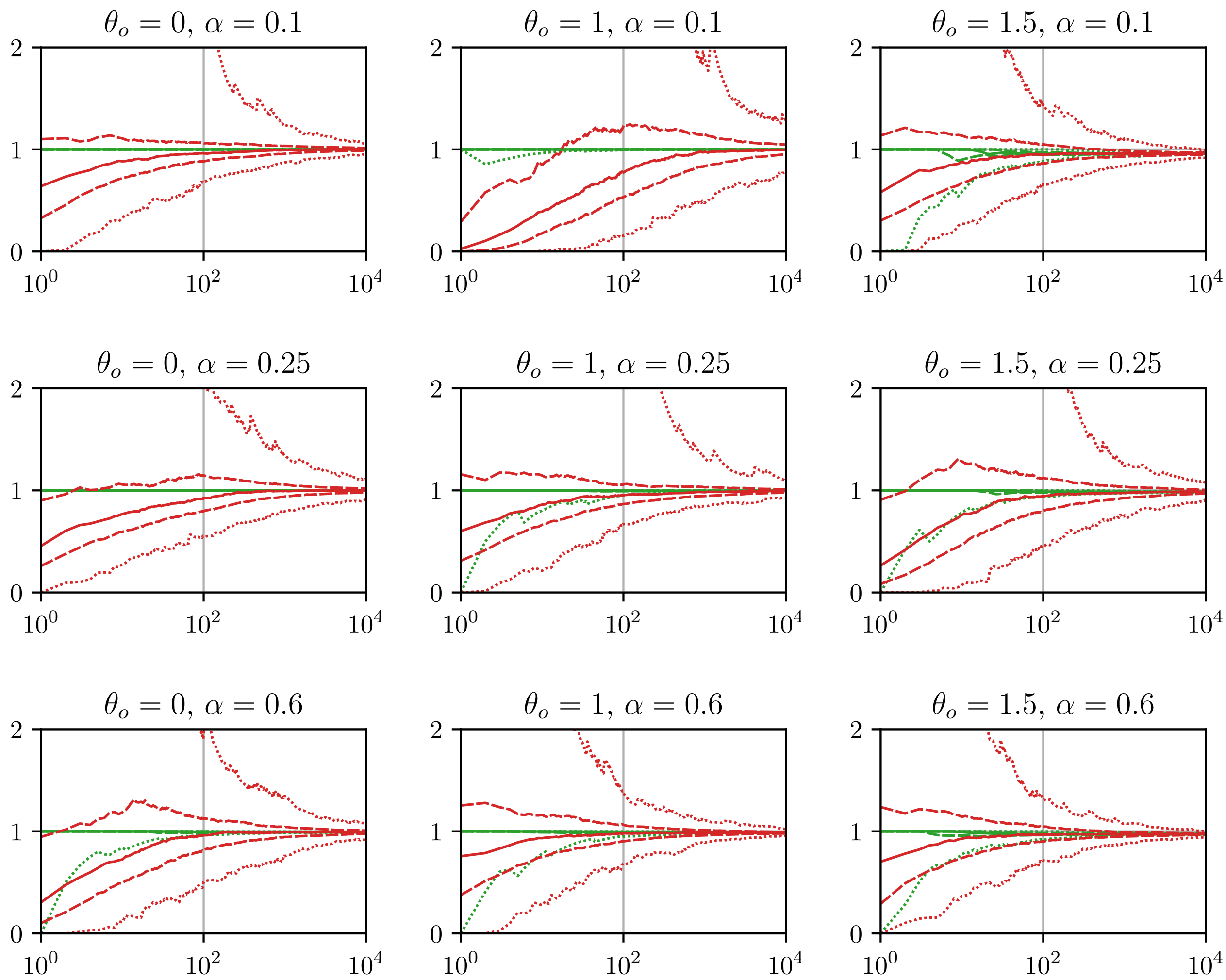

Figure 1: $K = 15$ microfacets within the pixel footprint.

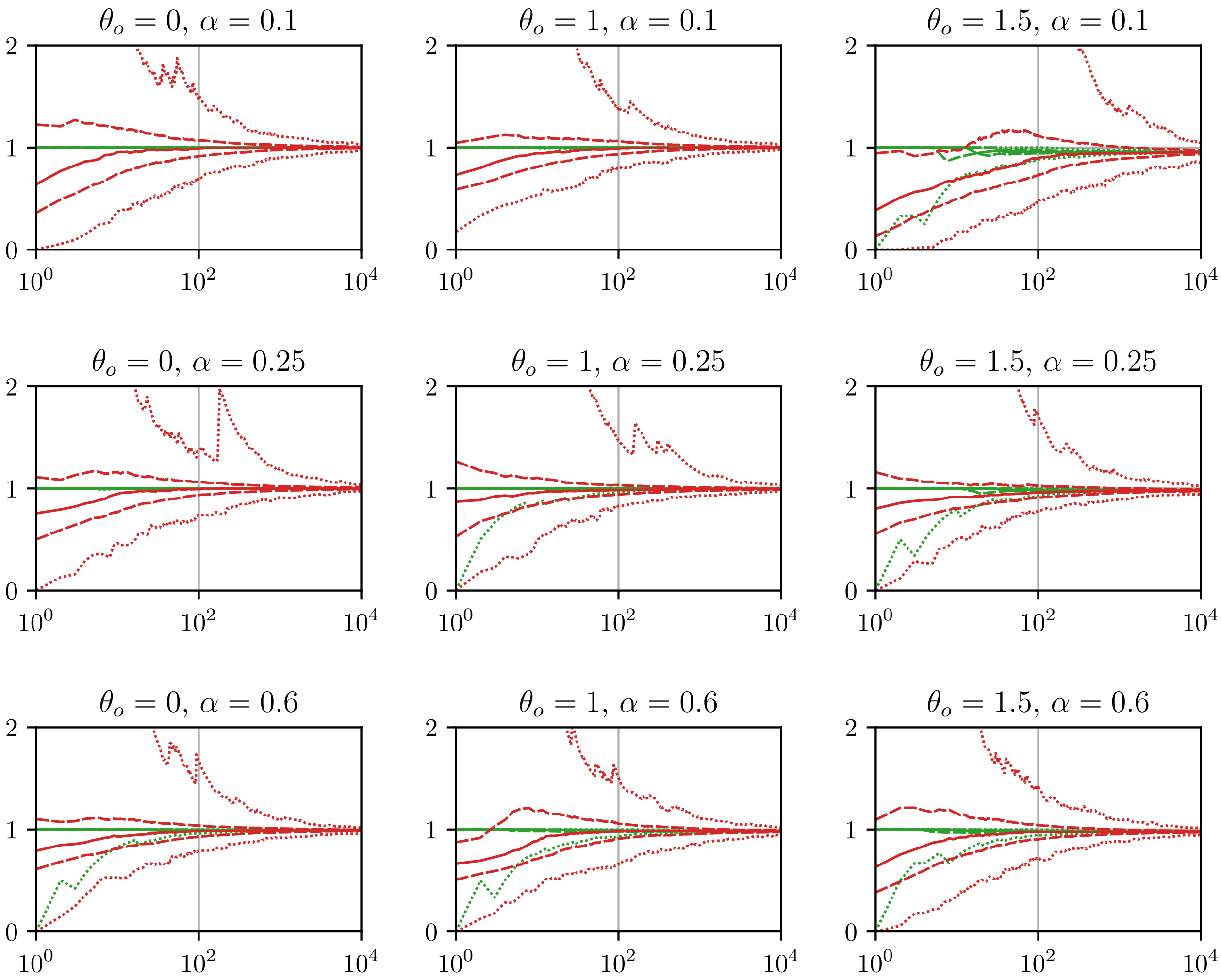

Figure 2: $K = 148$ microfacets within the pixel footprint.

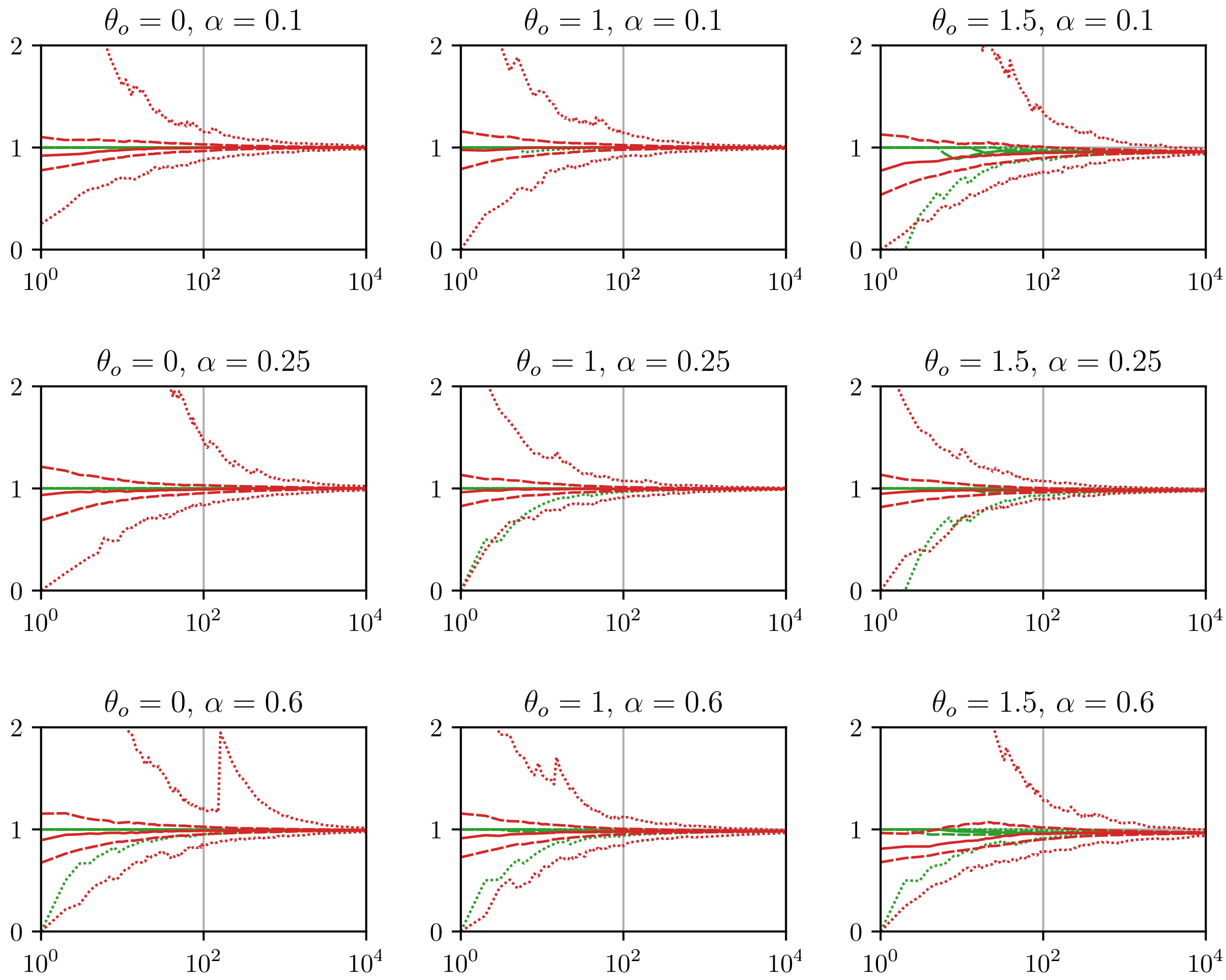

Figure 3: $K = 2,379$ microfacets within the pixel footprint.

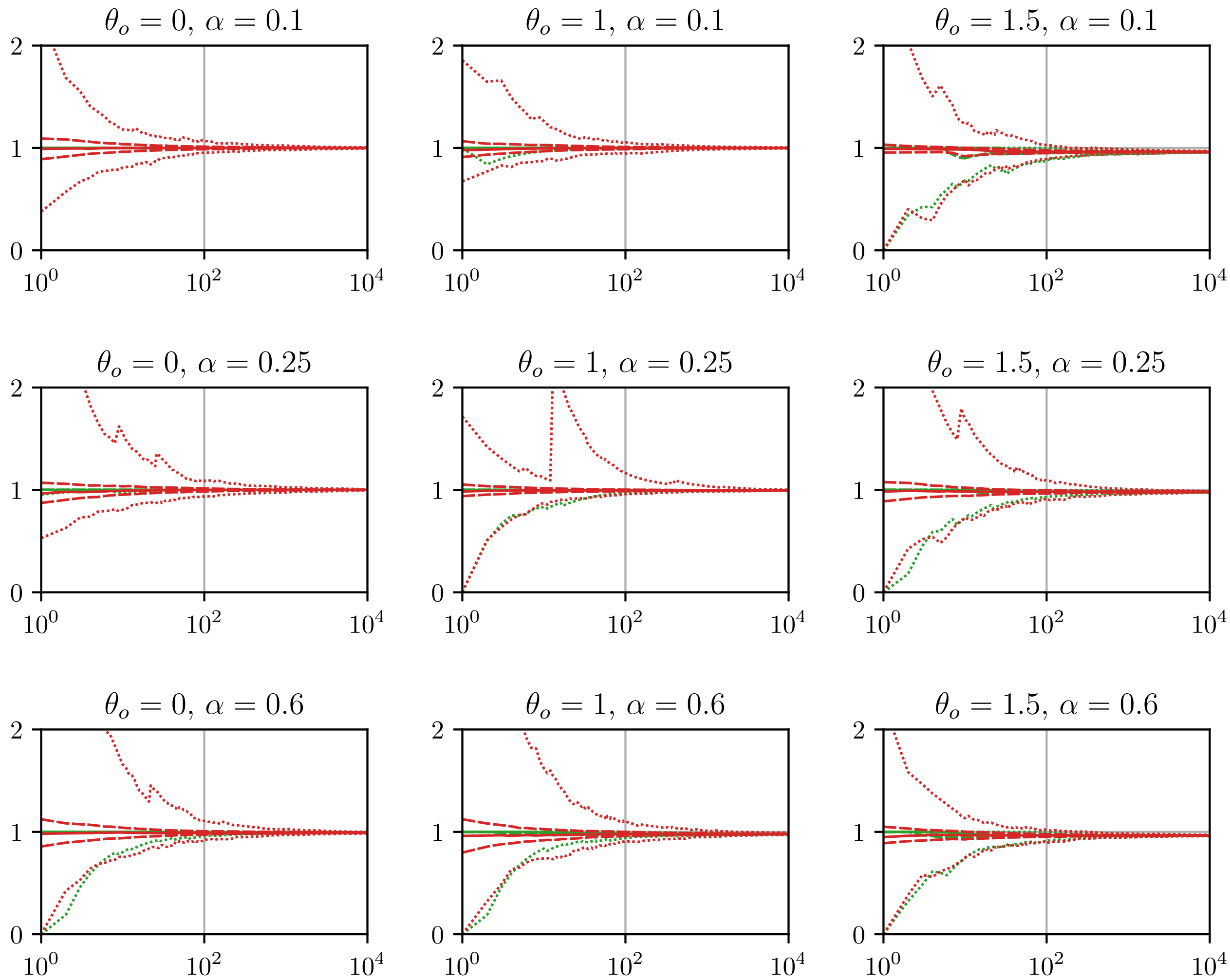

Figure 4: $K = 41,624$ microfacets within the pixel footprint

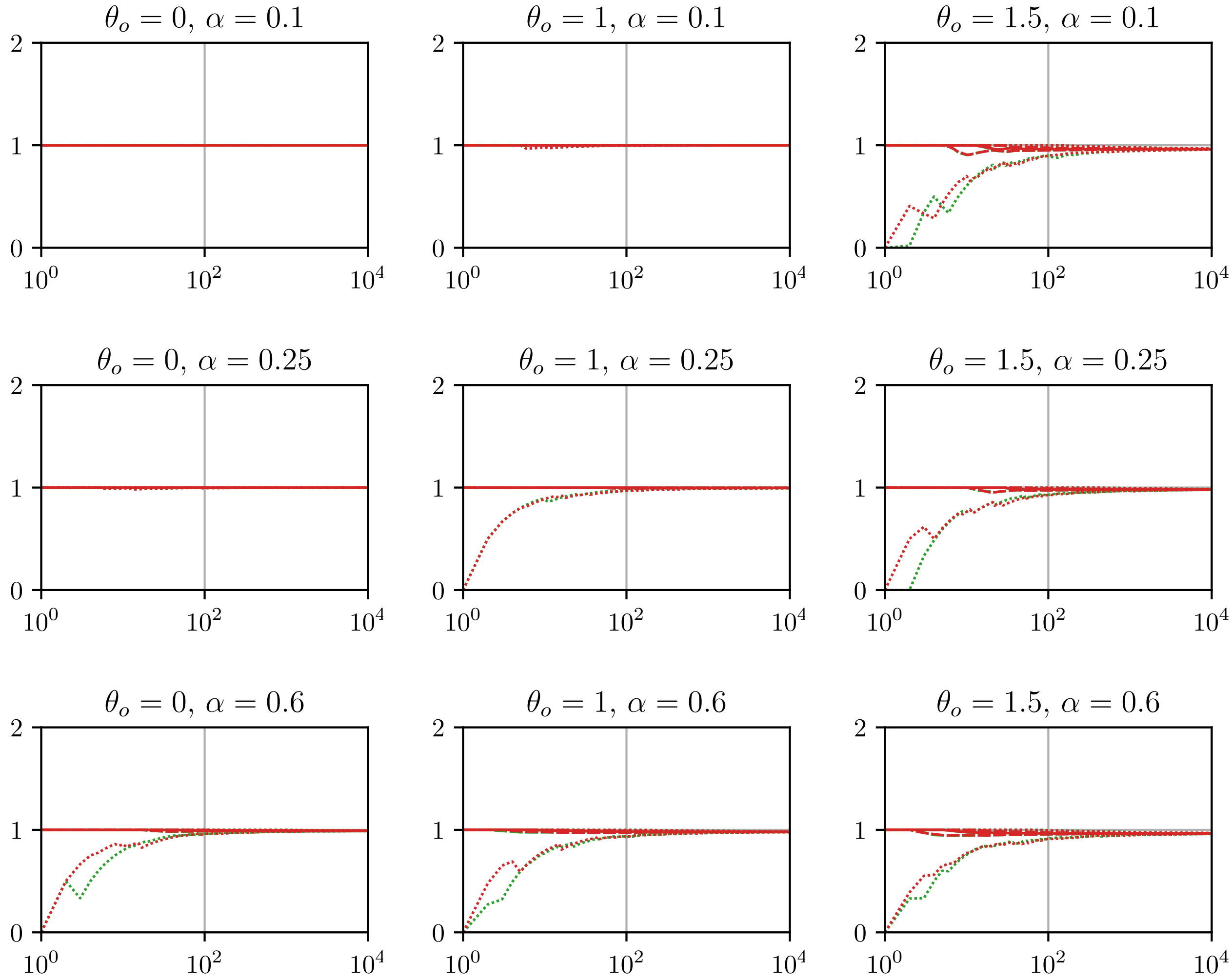

Figure 5: $K = 166,496$ microfacets within the pixel footprint. Here, the glittering NDF has converged and is a Gaussian. The last level of detail is reached, and there are no more glints. In both cases, the sampled PDF is the same, and this PDF has a shape very close to the BSDF.